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Monte Carlo Study of Tricritical Dynamics in Two Dimensions

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The dynamical tricritical behavior for the spin-1 Ising model with single-ion interaction is investigated in two dimensions using Monte Carlo simulations. The nonlinear dynamical tricritical exponent z_t is determined from the asymptotic power-law relaxation of the magnetization. The value $z_t = 1.99 \pm 0.04$ reported here is the first estimate of the dynamical exponent at a multicritical point, in two dimensions.

KEY WORDS: Tricritical dynamics; Monte Carlo; Blume–Capel model; dynamic scaling; tricritical exponents; nonlinear relaxation.

1. INTRODUCTION

Static tricritical properties of magnetic systems have been investigated in two and three dimensions using various techniques.[(1–10),3] For dimensionalities $d \geqslant 3$ the tricritical exponents are mean-field-like except for logarithmic corrections at $d = 3$. For $d = 2$ strong fluctuations are expected to change the critical exponents. In some cases, as in the next-nearest-neighbor Ising antiferromagnetic,[(11)] even the qualitative features of the phase diagram predicted by mean field theory are modified by the fluctuations. Very little, however, is known about the dynamical behavior at a tricritical point.[(1,2)] Early work in tricritical dynamics includes the study of the tricritical relaxation of the 3D Ising model with competing interactions.[(2)] In that work, however, no attempt was made to give an accurate estimate of the dynamic exponent. We have therefore carried out a Monte Carlo simulation in order to study the time evolution of the order

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[3] See Ref. 1 for a review.

parameter at a tricritical point with Glauber dynamics and to estimate the dynamic exponent associated with the relaxation process. We have chosen the Blume–Capel model.

The Blume–Capel model is a spin-1 Ising model in the presence of a single-ion anisotropy, namely

$$H = -J \sum_{\langle ij \rangle} S_i S_j + D \sum_i S_i^2 - H \sum_i S_i \tag{1}$$

where the Ising ferromagnetic interactions are restricted to the Z nearest neighbor pairs of spins. In the absence of the magnetic field H, the crystal field interaction raises the energy of the $S = \pm 1$ state by an amount D above the $S = 0$ state. In the limit of very strong anisotropy ($D \to \infty$) the $S = 0$ state is no longer occupied and the model reduces to the usual spin-1/2 model. The static critical and tricritical properties of the BC model have been investigated in detail by mean field theory,[3] renormalization group methods,[4–6] Monte Carlo simulations,[7,8] series expansions,[9] and Monte Carlo renormalization group analysis.[10]

A mean field approximation for the BC model predicts a second-order phase transition for the critical line with a critical temperature

$$k_B T_c / J = 2Z / [2 + \exp(D / k_B T_c)]$$

This line remains second order for positive values of the single-ion anisotropy up to the tricritical point at $k_B T_c / ZJ = 1/3$ and $D/ZJ = \frac{2}{3} \ln 2$. Beyond this point the critical line becomes first order for increasing values of the anisotropy, but for $D/ZJ > 1/2$ no phase transition occurs. As one would expect from universality arguments, the static critical exponents along the critical line remain Ising-like until the tricritical point is reached and a crossover phenomenon takes place, changing discontinously the critical exponents to tricritical ones.

Although the mean-field picture is qualitatively correct, a precise location of the critical line and particularly of the tricritical point (which is essential for the study of critical dynamics) is a difficult task. Very recently, however, accurate numerical results from Monte Carlo renormalization group[11] and transfer matrix[12] studies have become available on these topics. Early Monte Carlo studies of the bulk properties of the BC model[7] apparently overestimated the value of the tricritical temperature, as did recent MC analysis of interface behavior[13] for the same model.

2. METHOD

The dynamic finite-size scaling theory[14] predicts for the time evolution of the magnetization of a system of linear size L and dimension d the following scaling relation:

$$M(t, \varepsilon, g, h, L) = L^{-\beta_t/\nu_t}\tilde{M}(tL^{-z_t}, \varepsilon L^{1/\nu_t}, gL^{\phi_t/\nu_t}, hL^{\beta_t\delta_t/\nu_t}) \tag{2}$$

where ε and h are the temperature and magnetic scaling fields, respectively. The scaling field g measures the deviation from the tricritical point along the line tangent to the transition line at the tricritical point.[1,2,12] The exponents β_t, ν_t, ϕ_t, and δ_t are the static tricritical exponents and z_t the dynamical one. At tricriticality $\varepsilon = g = h = 0$ and the relation (2) is reduced to the form

$$M(t, L) = L^{-\beta_t/\nu_t}\tilde{M}(tL^{-z_t}) \tag{3}$$

This expression implies the existence of a crossover time $t^* \sim L^z$ such that for times below t^* a bulklike power-law behavior for the magnetization takes place and for $t > t^*$ finite-size effects are observed, giving rise to a exponential decay for the magnetization. Since we are making simulations in large ($L = 1200$) systems for relatively short times, the magnetization will behave like

$$M \sim t^{-\beta_t/\nu_t z_t} \tag{4}$$

The above expression can be used to give direct information on the dynamical tricritical exponent z_t.

The Monte Carlo method[15] has been used to perform simulations on the Hamiltonian (1) at the tricritical point using a 1200×1200 lattice with periodic boundary conditions. We use the value $k_B T_t/J = 0.6091 \pm 0.0030$ and $D_t/J = 1.9655 \pm 0.0151$ for the location of the tricritical point, recently obtained through a Monte Carlo renormalization group technique.[11] The spins evolve from a nonequilibrium configuration, where all spins are in the $S = 1$ state, to equilibrium through Glauber dynamics. The spins are flipped with the transition probability

$$p = \exp(-\Delta E/k_B T_t)/[1 + \exp(-\Delta E/k_B T_t)]$$

where ΔE denotes the change in the energy for a spin flip. To allow simulation for large lattices we have applied multispin coding.[16]

3. RESULTS

In Fig. 1 we show the time evolution of the magnetization. The time is measured in Monte Carlo steps per spin (MCS). Each point represents an

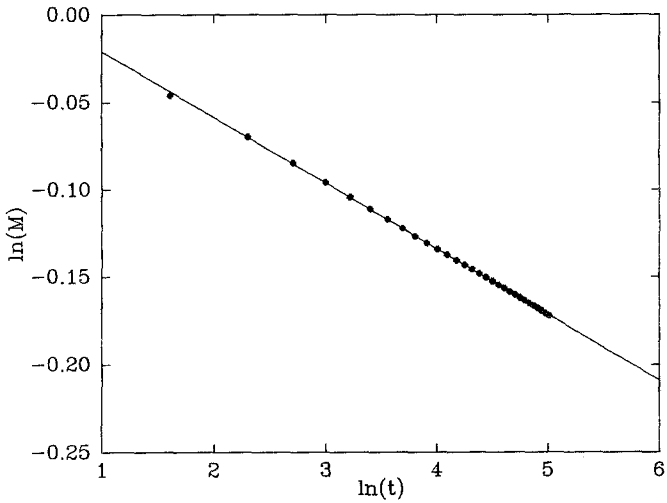

Fig. 1. Time evolution of the magnetization. (●) The Monte Carlo data; (—) the linear best fit in the interval $50 \leqslant t \leqslant 150$.

average over 40 samples. Measurements were made every 5 MCS. The error in each measurement is smaller than the size of the point. The straight line with slope $-\beta_t/\nu_t z_t = -0.03767 \pm 0.00073$ was obtained by an error-weighted least square fit for times between 50 and 150 MCS. The data show the expected power-law behavior for the magnetization for times as short as 10 MCS, indicating that the above interval used to obtain the slope is well inside the asymptotic region. The data also show that the magnetization relaxes toward the equilibrium at a much slower rate at the tricritical point than at a critical one.

In order to show that the limits large L and large t are reached, we also performed simulations using 160×160 and 320×320 lattices. The time evolution of the magnetization was recorded up to 500 MCS. The results are shown in Fig. 2. Measurements were made every 10 MCS. Each point represents an average over 600 and 100 samples for the small and large lattices, respectively. The straight line is the same obtained in Fig. 1. As one can see, no lattice effects are observed and the magnetization decay is not changing on a scale of 500 MCS, suggesting that the above limits are reached.

To obtain an explicit value for z_t we use the conjectured values[17] for the static tricritical exponents $\beta_t = 1/24$ and $\nu_t = 5/9$. Recent Monte Carlo renormalization group results[11] suggest that the above conjecture might be in fact exact. This gives for the dynamic tricritical exponent the value $z_t = 1.99 \pm 0.04$. To the best of our knowledge, this is the first attempt to

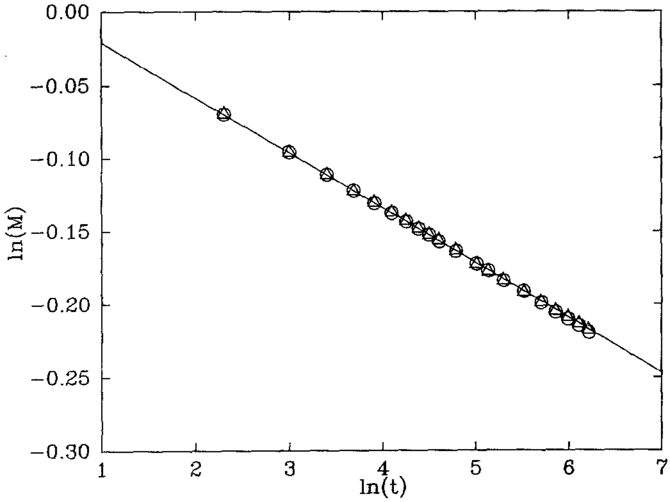

Fig. 2. Time evolution of the magnetization for times up to 500 MCS. The lattices sizes are (○) 160 × 160 and (△) 320 × 320. (—) The same obtained in Fig. 1.

give an accurate estimate of the nonlinear dynamical critical exponent at a multicritical point, in two dimensions. An analysis of the critical–tricritical dynamical crossover phenomenon will be presented elsewhere.

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